

# Dynamic Brand Choice Modeling Based on the State-space Approach

Eiji Motohashi      Second-year Doctoral Student, The Graduate University for Advanced Studies

## 1 Introduction

We develop a dynamic brand choice model incorporating consumer heterogeneity and market dynamics based on the state space model (Figure 1). Then we apply the proposed model to scanner panel data to capture market trend and to predict consumer behavior.

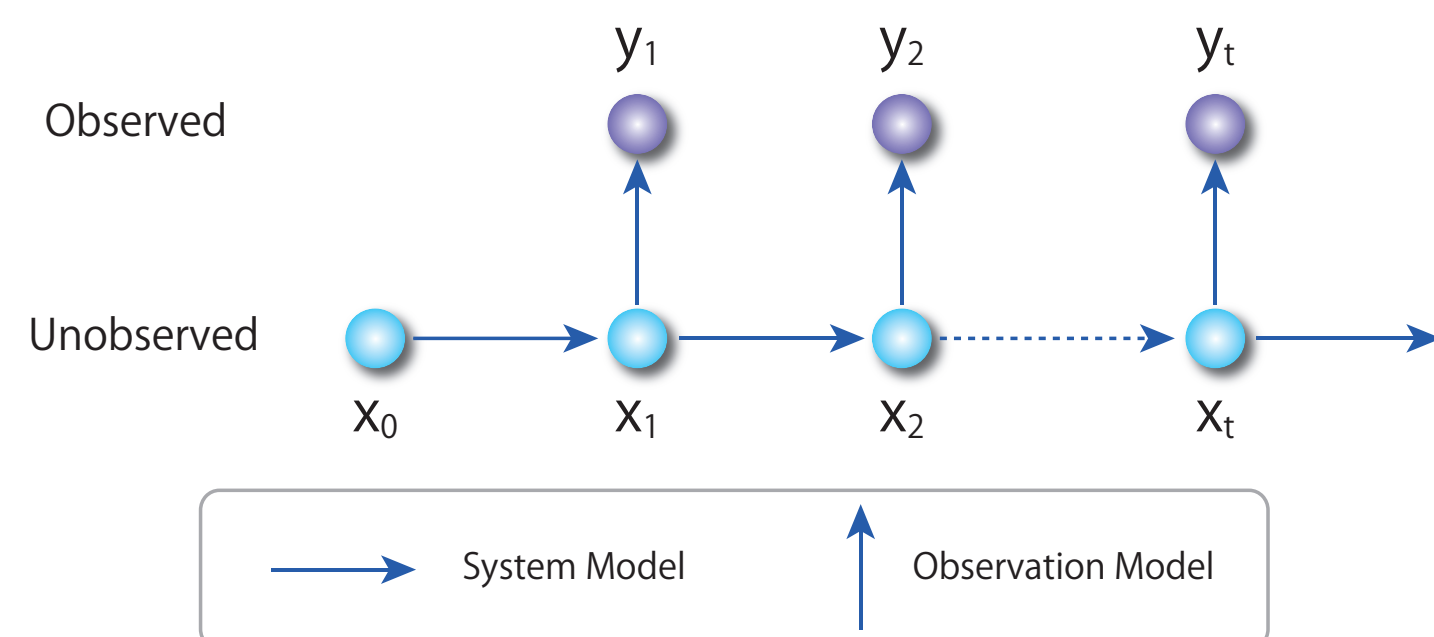


Figure 1: State Space Model

## 2 Model

The proposed model can be considered within the framework of the state space model. We use the multinomial logit model as an observation model to represent how consumers make brand choice decisions, and represent mechanism of parameter changes in a system model. We model the probability that household  $k$  purchase brand  $i$  at time  $t$ , given category purchase, as

$$P(Y_{ik}^t = 1) = \frac{\exp(X_{ik}^t \beta_k^t + \eta_k \mathbb{I}\{s_k^t = i\})}{\sum_j \exp(X_{jk}^t \beta_k^t + \eta_k \mathbb{I}\{s_k^t = j\})}, \quad (1)$$

where  $X_{ik}^t$  is a row vector of explanatory variables for brand  $i$  of household  $k$  at time  $t$ ,  $\beta_k^t$  is an  $m$  dimensional column vector that consists of state vector and time-invariant parameters, and  $s_k^t \in \{1, \dots, I\}$  indicates the household  $k$ 's purchased brand at the previous purchase occasion. Next, we model the individual heterogeneity of  $\beta_k^t$  as

$$\beta_k^t = \Psi^t + \Phi Z_k, \quad (2)$$

where  $\Psi^t$  is an  $m$  dimensional state vector,  $Z_k$  is an  $l$  dimensional column vector of household specific variables for household  $k$ , and  $\Phi$  is an  $m \times l$  time-invariant parameter matrix to be estimated. Likewise, we formulate  $\eta_k$  as

$$\eta_k = \gamma + \Upsilon Z_k, \quad (3)$$

where  $\gamma$  is a parameter and  $\Upsilon$  is an  $l$  dimensional parameter vector. Note that  $\gamma$  and  $\Upsilon$  are both time-invariant. Finally, we model the system model that represents the dynamic change of the state vector,  $\Psi^t$ , as

$$\Psi^t = \Psi^{t-1} + \Xi^t, \quad \Xi^t \sim N(0, \Sigma), \quad (4)$$

where  $\Sigma$  is a variance-covariance matrix of  $\Xi$ . That is, the system equation simply follows the random walk model.

## 3 Empirical Analysis

### 3.1 Data

To estimate the proposed model, we use scanner panel data recorded at a super market in Japan. The product category is instant coffee and the length of the data period is three years (1,067 business days) from the beginning of 2000 to the end of 2002.

### 3.2 Variable Specification

We use the following information for the explanatory variables and household specific variables in the proposed model.

**Explanatory Variables:**  $X_{ik}^t$

- $PRICE_i^t$ : the minimum price per gram for brand  $i$  at time  $t$  (there are several products of different size in the same brand)
- $DISPLAY_i^t$ : a binary variable indicating if at least one product of brand  $i$  is displayed at time  $t$
- $FEATURE_i^t$ : a binary variable indicating if at least one product of brand  $i$  is featured at time  $t$

**Household Specific Variables:**  $Z_k$

- $SPEND_k$ : three categories based on household  $k$ 's total purchase amount in the category divided by total purchase volume in the data period
- $FREQ_k$ : three categories based on household  $k$ 's total number of the category purchase divided by business days in the data period

### 3.3 Analysis Design

To examine the effectivity of incorporation of consumer heterogeneity and dynamic change in parameters, we compare the four models in Table 1.

Table 1: Comparative Models

	w/o Heterogeneity ( $\Phi = 0, \Upsilon = 0$ )	w/ Heterogeneity ( $\Phi \neq 0, \Upsilon \neq 0$ )
w/o Dynamic Change ( $\Sigma = 0$ )	Model 1	Model 2
w/ Dynamic Change ( $\Sigma \neq 0$ )	Model 3	Model 4

### 3.4 Results

Table 2 shows the log-likelihood of estimation and validation for each model. The log-likelihood of validation was calculated by using the estimated coefficients and smoothed distribution. From Table 2, we see that Model 4 best fits the data in estimation; however, Model 3 best fits the data in validation. This means that a model that considers only dynamic change in parameters is the best for prediction.

Table 2: Comparison of Log-likelihood

	Estimation	Validation
Model 1	-4812.78	-5086.09
Model 2	-4765.37	-5102.80
Model 3	-4803.65	-5077.72
Model 4	-4754.06	-5100.23

Figure 2 shows the transition of promotion effects obtained from the smoothed distribution. The time variation of price effect is relatively smaller than those of display and feature. We also see that the display effect in the first half is larger than those in the last half.

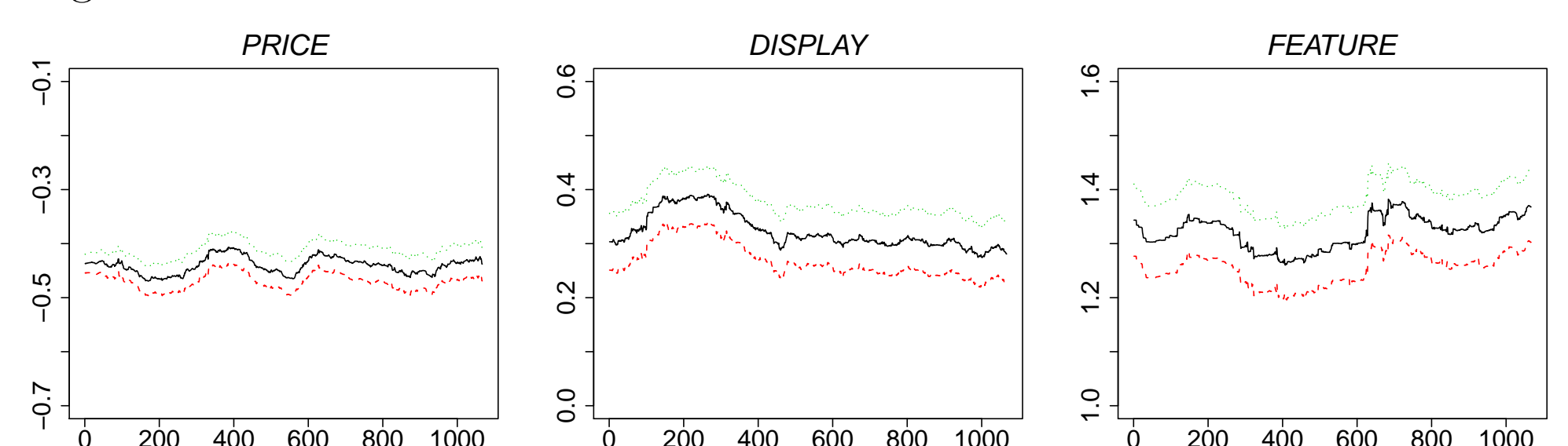


Figure 2: Transition of Promotion Effects